

Microcanonical treatment of black hole decay at the Large Hadron Collider

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Abstract. This study of corrections to the canonical picture of black hole decay in large extra dimensions examines the effects of back-reaction corrected and microcanonical emission at the LHC. We provide statistical interpretations of the different multiparticle number densities in terms of black hole decay to standard model particles. Provided new heavy particles of mass near the fundamental Planck scale are not discovered, differences between these corrections and thermal decay will be insignificant at the LHC.

1. Introduction

In higher-dimensional, low-scale gravity scenarios [1, 2, 3, 4, 5], black holes can be produced in high-energy particle collisions such as those anticipated from the Large Hadron Collider (LHC) [6, 7, 8]. Most of these models predict that black holes will rapidly decay in four successive phases of which Hawking evaporation [9, 10] may dominate. Accordingly, the Hawking evaporation phase has been the most studied decay phase in higher-dimensional models.

The thermodynamic description of black holes [11, 12, 13, 14] has been a powerful tool for probing quantum gravity aspects of black hole physics. The statistical mechanical interpretation of Hawking evaporation treats the black hole as a constant temperature reservoir that allows the emission of particles in thermal equilibrium with the black hole. While this is probably a good approximation for large black holes, its applicability to small primordial black holes or higher-dimensional black holes in low-scale gravity is not clear.

Modelling emissions with the canonical ensemble from these types of black holes is inappropriate since a black hole in asymptotically flat spacetime cannot be in stable thermal equilibrium with its radiation. Consequently, when the mass of the black hole is close to its temperature, its evaporation will be modified. The Hawking evaporation picture can be improved by applying the laws of statistical mechanics to ensure that the evaporation remains thermodynamically valid below the Planck scale.

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This can be done by using back-reaction corrected emission in a microcanonical ensemble [15, 16, 17, 18, 19, 20]. In addition, Ref. [21] has provided an alternative implementation of the back reaction of the emitted radiation through a modification of the relation between the black hole radius and its temperature.

The effects of quantum gravity should become important at the mass scales we will consider. Since the thermodynamic description of quantum systems is often a useful tool, the microcanonical description of black hole evaporation can be expected to remain valid in the quantum regime near the Planck scale.

This paper is organized as follows. Sec. 2 reviews Hawking evaporation. Following this, Sec. 3 introduces the microcanonical ensemble and back-reaction corrected emissions. It also discusses the regime in which the microcanonical description becomes important. Sec. 4 addresses the measurable particle energy spectra. Sec. 5 argues that, at the LHC, the microcanonical corrections will probably not be important. The paper concludes with a discussion of the applicability of the modified energy spectrum if new heavy particles are emitted from black holes.

2. Hawking Evaporation

Hawking radiation is not described by a pure quantum-mechanical state but by a density matrix. It is completely thermal in that the probabilities of emission of particles in different modes and probability of emitting different number of particles in the same mode are completely uncorrelated. The probability for different numbers of particles agree exactly with thermal radiation. It is assumed in Hawking evaporation that the black hole emits non-interacting particles.

The expectation value of the number of particles $\langle N \rangle$ of a given species emitted in a mode with frequency ω , angular momentum m about the axis of rotation of the black hole, and charge q is

$$\langle N \rangle = \frac{\Gamma(\omega)}{e^{2\pi(\omega - m\Omega - q\Phi)/\kappa} + s}, \quad (1)$$

where s is a statistics factor that is -1 for bosons and $+1$ for fermions; κ , Ω , and Φ are the surface gravity, surface angular frequency of rotation, and surface electrostatic potential; the species dependent $\Gamma(\omega)$ is the fraction of the mode that would be absorbed were it incident on the black hole. Since the expected number of particles emitted in each mode is the same as that of a thermal body whose absorptivity matches that of the black hole, the temperature of the black hole has been identified as $T = \kappa/(2\pi)$.

Hawking evaporation can be described by a number density. The number density represents the available states that can be occupied by Hawking radiation. For a non-rotating and non-charged black hole, the number density is most often represented by the canonical ensemble number density

$$n_T(\omega) = \frac{1}{e^{\omega/T} + s}, \quad (2)$$

where ω is the energy of the emitted particle (usually assumed to be massless) and the frequency-dependent (grey-body) factor Γ has been ignored for now. There are also charged and rotating black hole solutions, which contribute to the grand canonical ensemble in which the electronic potential and rate of rotation act as the chemical potential for charge and angular momentum [22].

The canonical ensemble is a hypothetical collection of systems of particles used to describe an actual individual system, which is in thermal contact with a heat reservoir but not allowed to exchange particles with its environment. The black hole is treated as a heat bath of fixed temperature and the back reaction of the particles on the black hole metric is neglected.

Because the temperature of a black hole increases as the mass decreases, it cannot be in stable thermal equilibrium. In addition, in asymptotically flat spacetime, black holes have a negative specific heat thereby implying that the canonical ensemble does not apply. Hawking emission of particle energies close to the black hole mass have non-negligible back reactions. This back reaction is responsible for modifying the temperature. Thus the heat-bath assumption breaks down as the mass of the black hole approaches the Planck scale. The back reaction of the metric has not yet been completely solved.

For black holes with mass of the order of the Planck scale, or less, the thermodynamical picture is no longer a good approximation of the true microcanonical description. We can make the above description valid from a thermodynamic point of view, even when the energy of the emitted particle is close to the black hole mass, by using the microcanonical ensemble.

The statistical mechanical description of black holes does not provide any time information. Two extreme views can be taken. At one extreme, all the emissions can be considered to occur so quickly that the black hole mass, and thus temperature, is approximately constant at its initial value during the decay. This is the so called “sudden approximation” view take by Dimopoulos and Landsberg [8]. The other extreme view is that the decay takes a long time and the black hole reaches thermal equilibrium after each emission before emitting the next particle. This quasistationary picture is adapted throughout this paper.

3. Microcanonical Ensemble

To describe black hole decay, we will employ the microcanonical ensemble of a large number of similar insulated systems each with a given fixed energy. Each of these systems will then have a number of different configurations compatible with the given energy. These configurations then form a shell in the configuration space of the system. As time passes, the system moves from one point to another in this shell. Then by the assumption of ergodicity, the probability of the whole system being a particular one in a chosen region of configuration space is proportional to the number of configurations in that region whose energies lie within an infinitesimal range.

The microcanonical ensemble number density for a single-particle microstate is given by the exponential of the change in black hole entropy before and after the emission

$$n_{\text{SP}}(\omega) = e^{-\Delta S} = \frac{e^{S(M-\omega)}}{e^{S(M)}}, \quad (3)$$

where $S(M)$ is the entropy of the black hole with initial mass M and $S(M - \omega)$ the entropy of the black hole after emitting a particle of energy ω . This single-particle distribution can be understood by interpreting the occupation of states as arising from a tunnelling probability [23]. The distribution has also been derived as the emission rate from excited D -branes using the microcanonical ensemble with back-reaction corrected emission rates in field theory [24, 25].

From the single-particle number density, the average particle density can be obtained by counting the multiplicity of states according to their statistics [26]:

$$n_{\text{BR}}(\omega) = \frac{1}{e^{S(M)-S(M-\omega)} + s}, \quad (4)$$

where $\omega < M$. This distribution approaches the thermal distribution (2) for $\omega \ll M$. The number density has similarities to an ideal gas of thermal radiation in equilibrium with a fix-temperature heat bath. The ω/T argument in the exponential of the canonical multiparticle number density has essentially been replaced by ΔS . ΔS has the nice property of always being bounded between 0 and 1. ΔS is the change in entropy of the black hole due to a single particle being in a state with mode energy ω . The k -th particle state is defined by a change in entropy, not in energy value. A k -particle state changes the entropy by $k\Delta S$, but does not have an energy $k\omega$. For fixed M and ω , the change in entropy is the constant quantity that sets the spacing between adjacent modes. More than one particle in a state must be viewed as moving to that state simultaneously before the black hole mass can change. For a black hole, the mass and entropy cannot change in such a way as to keep the temperature fixed. In other words, the mass and temperature of the black hole change in such a way as to keep ΔS constant. The energy of the black hole plus particle system is conserved. Thus the distribution represents the back-reaction corrected number density of an ideal gas. We refer to (4) as the multiparticle distribution for back-reaction corrected emissions in an ensemble of particle-occupied excited modes. The ideal gas analogy should not be pushed too far as the equivalent q or ψ function to the canonical or grand canonical ideal gas do not allow a straight forward determination of the macroscopic properties of the gas.

Equation (4) is not the only possibility for the multiparticle number density. By considering a black hole as an extended quantum object (p -brane), which is made of other black holes, and then considering a gas of p -brane black holes [27, 28], the occupation number density for the Hawking particles in the microcanonical ensemble has been proposed as

$$n_E(\omega) = \sum_{k=1}^{\infty} \frac{\Omega(E - k\omega)}{\Omega(E)} \Theta(E - k\omega). \quad (5)$$

In thermodynamic equilibrium, the statistical mechanical density of states is $\Omega(M) = e^{S(M)}$. By identifying the total energy of the system with the initial black hole mass, we write

$$n_E(\omega) = \sum_{k=1}^{\lfloor \frac{M}{\omega} \rfloor} e^{S(M-k\omega)-S(M)}, \quad (6)$$

where $\lfloor M/\omega \rfloor$ denotes the integer part of M/ω . This distribution approaches (4) and thus the thermal distribution (2) for $\omega \ll M$. Now each k -particle state is defined by a mode energy and changes the black hole entropy by a different amount. The number of particles in a state is not determined by the particle statistics but rather by truncating the sum appropriately to conserve energy. Each term can be viewed as a state but the number of particles in each state is not fixed. For example, one particle in a state with energy $k\omega$ is the same as k particles in a state with energy ω . Since each term can be viewed as a different energy of emission, and thus entropy change, ω represents the quanta of energy or mode energy. When plotting $n_E(\omega)$ versus ω we are allowing the mode energy ω to vary. The plot built up by scanning through the mode energies is the same as scanning through the particle energies. We refer to (6) as the multiparticle distribution of back-reaction corrected emissions in a microcanonical ensemble of p -brane defined black holes. Since (6) does not include the emitted particle's statistics factor s , it must be added by hand or the equation can only be applied for large changes in entropy, where the evaporation is governed by Boltzmann statistics.

Equation (6) has been used to calculate the black hole lifetime [19, 20, 29]. In four dimensions, the evaporation rate dM/dt diverges at $M = 0$ if the canonical number density (2) is used. The canonical ensemble justifies the use of the sudden approximation. On the other hand, using the microcanonical number density (6) leads to a finite decay rate and gives a lifetime for black holes with $M = 2M_P$ that is 10^9 times longer than that given by the canonical number density [19]. In higher dimensions, dM/dt is finite and slows down in the later stage of evaporation. The microcanonical ensemble justifies the use of the quasistationary approximation.

We can understand the relationship between (4) and (6) a bit better by restoring the units in (6) (as pointed out by Casadio and Harms [27]) to get

$$n_E(\omega) = \sum_{k=1}^{\lfloor \frac{Mc^2}{\hbar\omega} \rfloor} e^{S(Mc^2-k\hbar\omega)-S(Mc^2)}. \quad (7)$$

In the classical limit for fixed c and G_D , $\hbar \rightarrow 0$ and the upper limit in the sum $Mc^2/\hbar\omega \rightarrow \infty$. Only the lowest-order term proportional to ω/T remains when expanding the argument of the exponential. Since $M_P \propto (\hbar c/G_D)^{1/(n+2)} \rightarrow 0$, $\hbar \rightarrow 0$ is equivalent to $M/M_P \rightarrow \infty$. Thus the finite sum becomes important at black holes masses near the Planck scale, and (6) can be considered the quantum version of (4).

To compare the number densities, expand the entropy to leading order in ω/M to obtain

$$\begin{aligned}
S(M - \omega) &= S(M) - \frac{\omega}{T} - \frac{1}{2C_V} \left(\frac{\omega}{T}\right)^2 + \dots \\
&\approx S(M) - \frac{\omega}{T},
\end{aligned} \tag{8}$$

where C_V is the black hole specific heat. Thus for $\omega \ll M$, the microcanonical number density approaches the back-reaction corrected density and canonical density.

To examine the higher-order contributions, we need to write down the entropy expressions. For a black hole of mass M and horizon radius R in n extra-dimensional asymptotically flat spacetime, the entropy is

$$S(M) = \frac{4\pi}{n+2} R M = \frac{n+1}{n+2} \frac{M}{T} \tag{9}$$

and the entropy differences is

$$\Delta S = S(M) - S(M - \omega) = S(M) \left[1 - \left(1 - \frac{\omega}{M} \right)^{\frac{n+2}{n+1}} \right]. \tag{10}$$

Assuming the maximum difference between the distributions occurs at the highest energy emissions (in the kinematic limit $\omega \rightarrow M/2$)

$$(\Delta S)_{\max} \approx \frac{\omega}{T} - \frac{M}{2T} \left\{ 1 - 2^{\frac{n+1}{n+2}} \left[1 - \left(\frac{1}{2} \right)^{\frac{n+2}{n+1}} \right] \right\}. \tag{11}$$

The biggest difference from a thermal distribution occurs at low values of n :

$$(\Delta S)_{\max} \approx \frac{\omega}{T} - 0.05 \frac{M}{T} \quad \text{for } n = 2. \tag{12}$$

For large M/T (large S),

$$n_T(\omega) \rightarrow e^{0.05M/T} n_{\text{BR}}(\omega). \tag{13}$$

The exponential in $n_{\text{BR}}(\omega)$ dies quickly so the correction factor in the numerator does not have much effect. For small M/T , the exponential is of the same order as $s = \pm 1$. In this case, the correction factor will have a significant effect.

4. Particle Energy Spectrum

Experimentally, we measure the energy spectra for different particle types. The energy spectra can be predicted from the number density as follows. The particle emission rate from a non-rotating and non-charged black hole in three dimensions is

$$\frac{dN}{dt} \propto n(\omega) \frac{d^3k}{(2\pi)^3}. \tag{14}$$

For isotropic massless particle emission,

$$\frac{dN}{d\omega} \propto \omega^2 n(\omega). \tag{15}$$

Often we consider the energy spectrum as a probability distribution and normalize (15) to unity over some region of energy. This is especially common for Monte Carlo generators like CHARYBDIS [30, 31].

In the rest frame of the black hole, conservation of energy-momentum requires a particle with mass m to be emitted with an energy ω in the range

$$m < \omega \leq \frac{M}{2} \left[1 + \left(\frac{m}{M} \right)^2 \right]. \quad (16)$$

The canonical distribution (2) does not respect energy-momentum conservation and any value of ω is allowed. Although the thermodynamic concept breaks down for $\omega \geq M$, the distribution does not enforce this condition. Throughout this paper, the minimum black hole mass is assumed to be close to a Planck scale of about 1 TeV. The exact value of the Planck scale is not too important since the black hole mass is expressed in terms of the Planck scale. However, the Planck scale can not be much higher than a few TeV if we are to observe black holes at the LHC. Assuming the heaviest particle continues to be the top quark at LHC energies, the largest value of the emitted particle energy will be only 3% above the value $M/2$. We thus neglect the particle mass and take the upper limit on the emitted particle energy to be $M/2$. This kinematic limit affects most of the decays and modifies the energy spectrum of emitted particles in the canonical ensemble [32].

The definition of the Planck scale is important when considering energies near the Planck scale. The PDG [33] definition of the Planck scale

$$M_D^{n+2} = \frac{(2\pi)^n}{8\pi G_D} \quad (17)$$

has been chosen for use throughout this paper. This definition causes the factor in the entropy that does not depend on the ratio of the black hole mass to Planck scale to increase monotonically with increasing number of dimensions. If the Dimopoulos and Landsberg definition is used, a minimum in the entropy factor occurs at $n = 3$ [34]. For the PDG definition of the Planck scale and seven extra dimensions, the black hole mass approaches its temperature when it is about 0.3 times the Planck scale. In the following, cases for $n = 2$ to 7 are studied, $n = 2$ is referred to as low dimension and $n = 7$ as high dimension.

5. Results

We now examine the affect the different number densities have on the particle energy spectra. Table 1 shows the maximum difference between the canonical and back-reaction corrected energy spectra for different dimensions for bosons. Table 2 shows the corresponding results for fermions. The maximum differences range from about 10% to 26% and occur at a black hole mass equal to the Planck scale for bosons and equal to about 1.3 times the Planck scale for fermions. The maximum differences occur at a temperature of about 225 GeV and an entropy of about 4. In all cases, except

for four dimensional black holes emitting bosons, the maximum difference occurs at temperatures well below the black hole mass.

Table 1. Maximum difference Δ between the canonical and back-reaction corrected energy spectra for bosons. M and T are in units of M_D and $M_D = 1$ TeV.

n	M	T	$S(M)$	$S(M/2)$	Δ	Δ (%)
2	0.987	0.264	2.80	1.11	0.0106	21
3	0.973	0.240	3.25	1.37	0.0067	17
4	0.981	0.230	3.55	1.55	0.0050	14
5	1.001	0.227	3.78	1.68	0.0041	12
6	1.023	0.227	3.94	1.78	0.0035	11
7	1.049	0.229	4.07	1.86	0.0031	10

Table 2. Maximum difference Δ between the canonical and back-reaction corrected energy spectra for fermions. M and T are in units of M_D and $M_D = 1$ TeV.

n	M	T	$S(M)$	$S(M/2)$	Δ	Δ (%)
2	1.394	0.235	4.44	1.76	0.0073	26
3	1.336	0.221	4.83	2.03	0.0048	21
4	1.323	0.217	5.08	2.21	0.0037	17
5	1.333	0.217	5.28	2.36	0.0030	14
6	1.352	0.218	5.42	2.45	0.0026	13
7	1.379	0.222	5.53	2.54	0.0024	11

For large entropy, the canonical and back-reaction corrected energy distributions are similar. For $M = 5M_D$, the distributions restrict the particle energies to be $\omega \lesssim 0.3M$, with about $\omega \approx 0.06M$ being the most probable value. As the black hole decays down to the Planck scale, we see notable differences between the canonical and back-reaction corrected distributions. Figure 1 shows the energy distributions as a function of ω/M using the canonical number density (2) and back-reaction corrected number density (4). The normalization of the curves has not been modified from (2) and (4). There is a significant difference between the distributions. The distributions extend over the entire kinematic range and deviate from each other at about $\omega \approx 0.2M$ for boson and $\omega \approx 0.4M$ for fermions. The biggest differences occur at $\omega = M$, with significant differences still occurring at $\omega = 0.5M$. The back-reaction corrected distributions (figure 1a)) for low dimensions favour energies at the highest kinematically allowed particle energies.

Interpreting the energy distributions as probability density functions, requires that the curves be normalized within the physically allowed region of $0 < \omega \leq M/2$, as shown in figure 2. There is now very little difference between the canonical and back-reaction corrected distributions after normalization. For comparison, the greybody distributions using the canonical number density have been included as the dotted lines [30, 35]. For

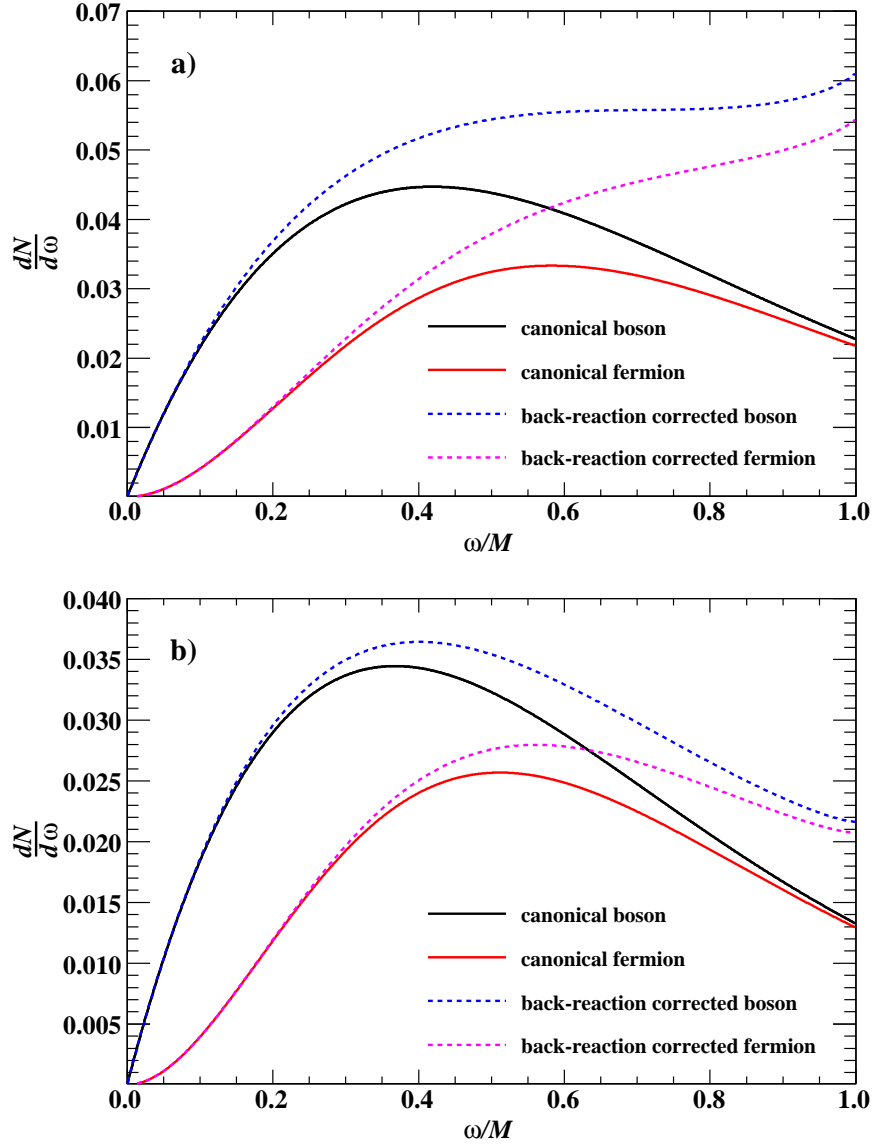


Figure 1. Energy spectra for $M = M_D = 1$ TeV. a) $n = 2$, $T = 260$ GeV, $S = 3$ and b) $n = 7$, $T = 230$ GeV, $S = 4$.

higher dimensions, the effect of the greybody factors is more significant than the back reaction correction. In all cases, the vector boson greybody factors give a significant difference. For the affects of greybody factors on high-entropy black holes see [36].

Identical distributions to those presented throughout this paper have been obtained using the black hole Monte Carlo event generator CHARYBDIS by restricting the mass of the black hole to an arbitrary range of 2 GeV around M and looking at the energy of the first emitted particle in the black hole rest frame. About 10^6 events are needed to reduce the statistical fluctuations to where the figures in this paper are effectively reproduced. Small discontinuities in the distributions are visible at the mass thresholds for the top quark and heavy gauge bosons.

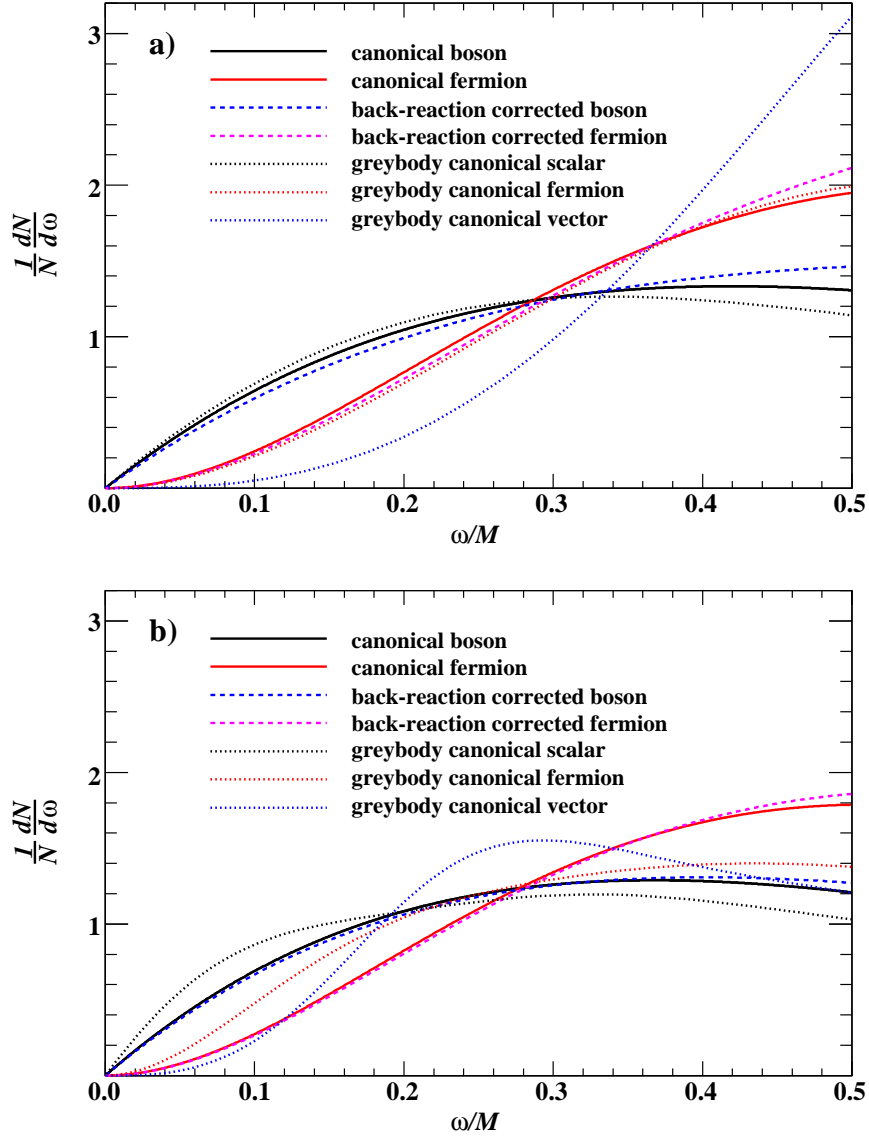


Figure 2. Normalized energy spectra for $M = M_D = 1$ TeV. a) $n = 2$, $T = 260$ GeV, $S = 3$ and b) $n = 7$, $T = 230$ GeV, $S = 4$.

Figure 3 shows energy distributions as a function of ω/M using the canonical number density (2) and microcanonical number density (6). For black hole masses near the Planck scale, the finite sum's impact becomes clear. The microcanonical distributions are again more similar to the canonical distributions after normalizing over the kinematic range of $\omega < M/2$, as shown in figure 3b). In future experiments, finite detector acceptance and resolution effects will most likely wash out the structure in the microcanonical distributions.

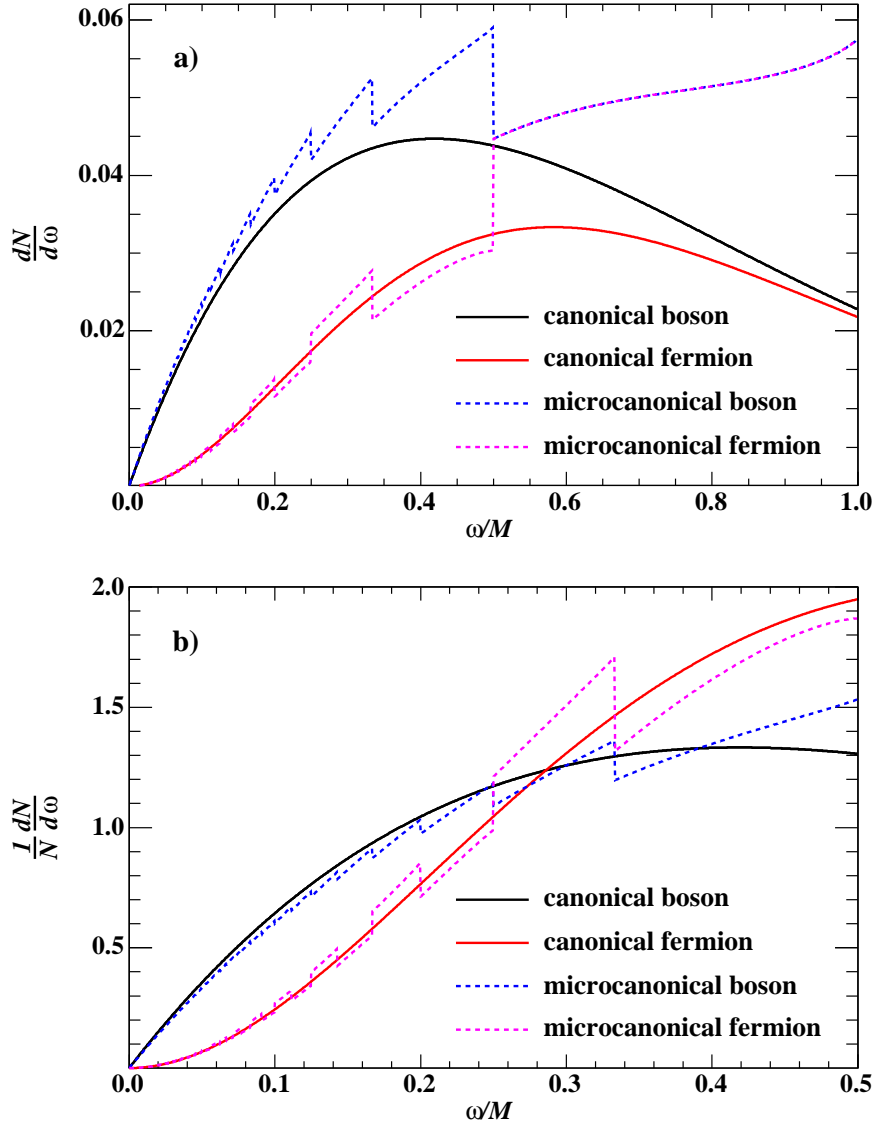


Figure 3. a) Energy spectra and b) normalized energy spectra for $M = M_D = 1$ TeV, $n = 2$, $T = 260$ GeV, $S = 3$.

6. Discussion

The six previous distributions are rather theoretical. At the LHC, the laboratory is unlikely to ever be in the rest frame of the black hole. Not boosting the particle energy spectra back into the rest frame of the black hole will wash out any signature of Hawking evaporation; boosting the particles is essential. Since the effects here appear near the Planck scale, the entire decay chain (time ordered) must be reconstructed or focus must be placed on low-mass black hole production. Studies of time ordering of black hole decay have been shown to not be very effective [30, 37]. In addition, at the end of a decay chain the black hole becomes highly boosted and thus the energy of particles

in the laboratory frame can be as high as M to $2M$. The other approach is to study only the first emitted particle from low-mass black holes. In this case, the multiplicities are low and we are immediately in the quantum gravity regime. Equivalent studies of first-emitted particles for high-mass black holes are far from trivial [30, 37]. Applying the techniques to black holes near the Planck scale should only be attempted with a realistic simulation of a detector.

The differences between the canonical and microcanonical energy distributions are very small over the allowed kinematic region for the Standard Model particles emitted by black holes above the Planck scale. Distinguishing between the ensembles will require an accurate determination of the black hole four-momentum in order to boost the particle energy into the black hole rest frame. Taking into account detector acceptance and resolution effects, distinguishing between the canonical and microcanonical distributions is unlikely to be possible at the LHC.

If a new heavy particle is discovered to decay from black holes at the LHC, it may be possible to identify the statistical ensemble provided the energy spectrum of the particle can be measured. For example, for a heavy particle of about half the Planck scale, the allowed kinematic region in black hole decay is $0.5 < \omega/M < 0.625$. In this region, the canonical and microcanonical energy distributions are significantly different and normalization over a narrow region of ω/M does not affect the difference. However, since decays to the heavy particle well above the Planck scale do not uniquely identify the ensemble distribution, the decays must be identified near the Planck scale, which could be problematic. In addition, the probability of emission of such a heavy particle would be small so it would be hard to accumulate reasonable statistics for such decays, even if they did occur.

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References

- [1] Arkani-Hamed N, Dimopoulos S and Dvali G 1998 *Phys. Lett. B* **429** 263
- [2] Antoniadis I, Arkani-Hamed N, Dimopoulos S and Dvali G 1998 *Phys. Lett. B* **436** 257
- [3] Arkani-Hamed N, Dimopoulos S and Dvali G 1999 *Phys. Rev. D* **59** 086004
- [4] Randall L and Sundrum R 1999 *Phys. Rev. Lett.* **83** 3370
- [5] Randall L and Sundrum R 1999 *Phys. Rev. Lett.* **83** 4690
- [6] Banks T and Fischler W 1999 *Preprint* hep-th/9906038
- [7] Giddings S B and Thomas S 2002 *Phys. Rev. D* **65** 056010
- [8] Dimopoulos S and Landsberg G 2001 *Phys. Rev. Lett.* **87** 161602
- [9] Hawking S W 1974 Black hole explosions? *Nature* **246** 30
- [10] Hawking S W 1975 *Commun. Math. Phys.* **43** 199
- [11] Bekenstein J D 1973 *Phys. Rev. D* **7** 2333
- [12] Bardeen J M, Carter B and Hawking S W 1973 *Commun. Math. Phys.* **31** 161
- [13] Bekenstein J D 1974 *Phys. Rev. D* **9** 3292

- [14] Hawking S W 1976 *Phys. Rev. D* **13** 191
- [15] Harms B C and Leblanc Y 1992 *Phys. Rev. D* **46** 2334
- [16] Harms B C and Leblanc Y 1993 *Phys. Rev. D* **47** 2438
- [17] Harms B C and Leblanc Y 1995 *Ann. Phys.* **244** 262
- [18] Harms B C and Leblanc Y 1995 *Ann. Phys.* **244** 272
- [19] Hossenfelder S 2005 What black holes can teach us *Focus on Black Hole Research*, Nova Science Publishers p155
- [20] Casadio R and Harms B 2001 *Phys. Rev. D* **64** 024016
- [21] Alberghi G L, Casadio R, and Tronconi A 2006 *J. Phys. G: Nucl. Part. Phys.* **34** 767
- [22] Gibbons G W and Hawking S W 1977 *Phys. Rev. D* **15** 2752
- [23] Parikh M K and Wilczek F 2000 *Phys. Rev. Lett.* **85** 5042
- [24] Keski-Vakkuri E and Kraus P 1997 *Nucl. Phys. B* **491** 249
- [25] Das S, Dasgupta A and Sarkar T. 1997 *Phys. Rev. D* **55** 7693
- [26] Koch B, Bleicher M and Hossenfelder S 2005 *J. High Energy Phys.* JHEP10(2005)053
- [27] Casadio R and Harms B 1998 *Phys. Rev. D* **58** 044014
- [28] Cirilo-Lombardo D J and Sanchez N G 2007 *Preprint* 0708.0393 [hep-th]
- [29] Casadio R and Harms B 2000 *Phys. Lett. B* **487** 209
- [30] Harris C M, Richardson P and Webber B R 2003 *J. High Energy Phys.* JHEP08(2003)033
- [31] Gingrich D M 2006 *Preprint* hep-ph/0610219
- [32] Harris C M, Palmer M J, Parker M A, Richardson P, Sabetfakhri A and Webber B R 2005 *J. High Energy Phys.* JHEP05(2005)053
- [33] Yao W-M *et al* (*Particle Data Group*) 2006 *J. Phys. G* **33** 1
- [34] Gingrich D M 2006 *Int. J. Mod. Phys. A* **21** 6653
- [35] Harris C M and Kanti R 2003 *J. High Energy Phys.* JHEP10(2003)014
- [36] Gingrich D M 2007 *J. High Energy Phys.* JHEP11(2007)064
- [37] Palmer M J 2005 Studies of Extra-Dimensional Models with the ATLAS Detector *PhD thesis*, University of Cambridge